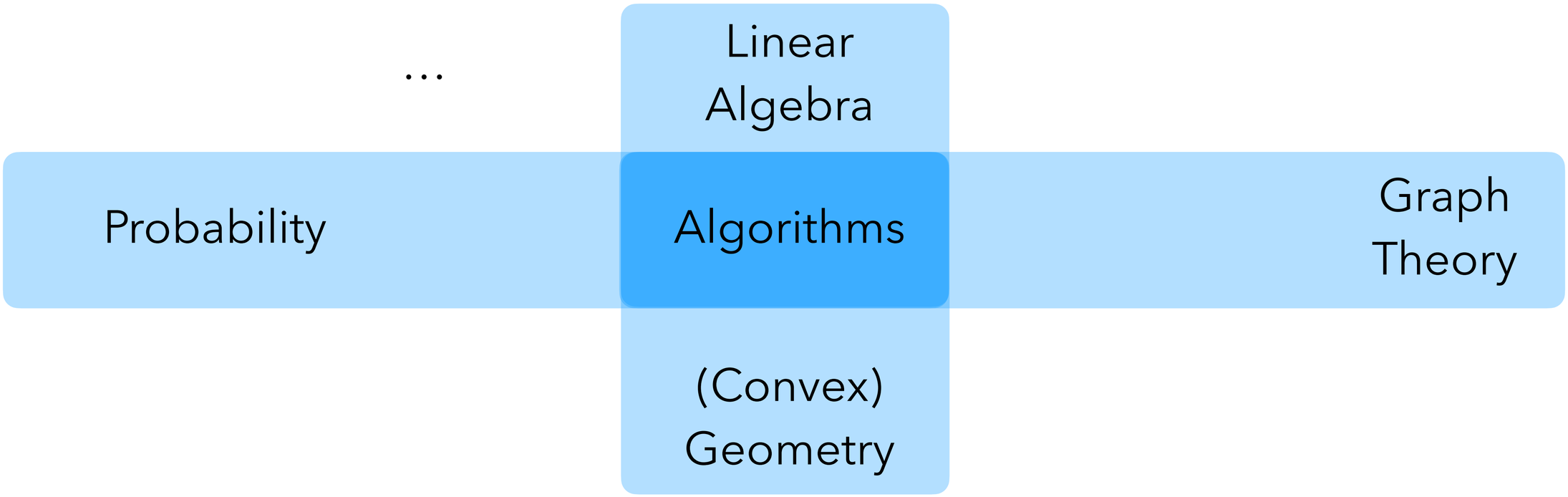
An Algorithmist's Toolkit

CSCI 2952T
Fall 2024
Brown University

D Ellis Hershkowitz (Ellis)

Class Overview

Class Overview



Class Overview

Main Goal: learn mathematical tools of modern algorithms research

Class Overview

Course Goals

- Jumpstart your research in algorithms
- Give you the algorithms toolkit to use in your research

  even if you're not a theory student

- Enable you to read algorithms papers / attend talks at

  Primarily: STOC/FOCS/SODA/SOSA/ICALP/ESA/ITCS

  Secondarily: EC/PODC/DISC/SPAA/COLT/NeurIPS+

- Teach you cool math

Class Overview

Disclaimers

- **This is a theory class**

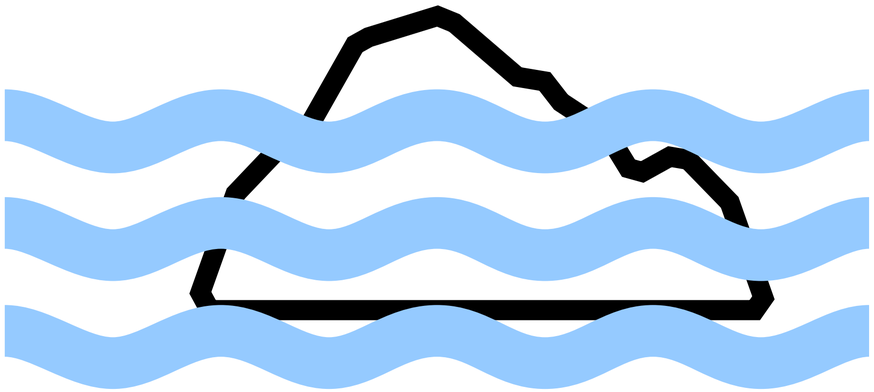
  entirely proof-based

- **This is biased towards my interests / research**

  there are some notably absent topics

- **This is a random walk**

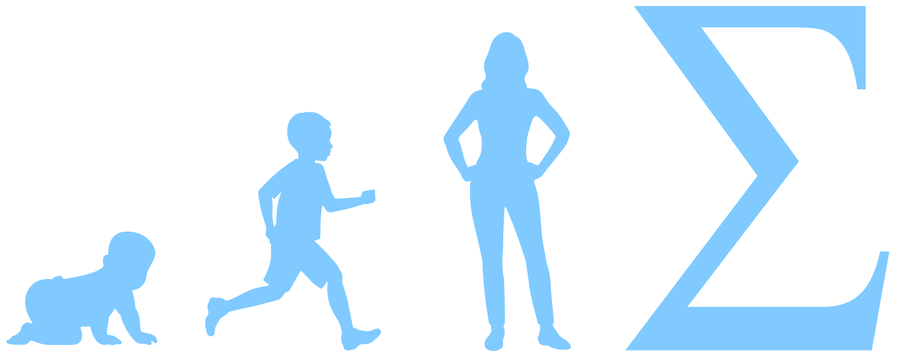
  breadth over depth (mostly)



Class Overview

Prereqs

- **Hard Requirements:**
  - ≥ 1 previous algorithms class (1570)
  - Mathematical maturity
- **Soft Requirements;** some familiarity with:
  - Probability
  - Linear algebra
  - Complexity (basics of NP-completeness)
  - A bit of calculus



Ask Me If Unsure!

Topics Overview

Topics Overview

Doing Theory is Hard

The right tools make theory easier:

1. **Basic Tools**
2. **Randomized Algorithms**
3. **Polyhedral Methods**
4. **Geometric Methods**
5. **Cuts and Flows**
6. **Graph Sparsification**
7. **Multiplicative Weights**

Topics Overview

Basic Tools

Squinting at **symbols** makes them easier to think about

Topics Overview

Basic Tools

$$100 \cdot \log^3 n \cdot \sqrt{n} + \frac{\log^{12} n}{\log \log n} \cdot \exp(10 \cdot \log n) = n^{10+o(1)}$$

Asymptotics

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \leq O(\log n)$$

Inequalities

How to Push Symbols Around Like a Pro

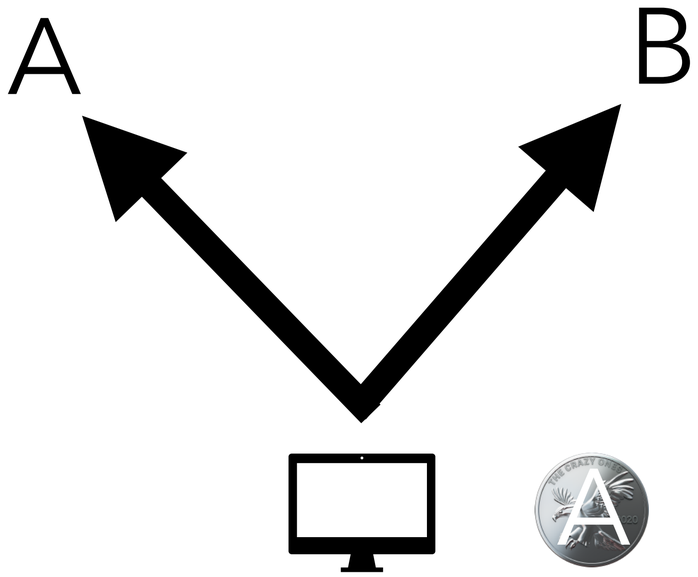
Topics Overview

Randomized Algorithm

Making **random** decisions makes decisions easier

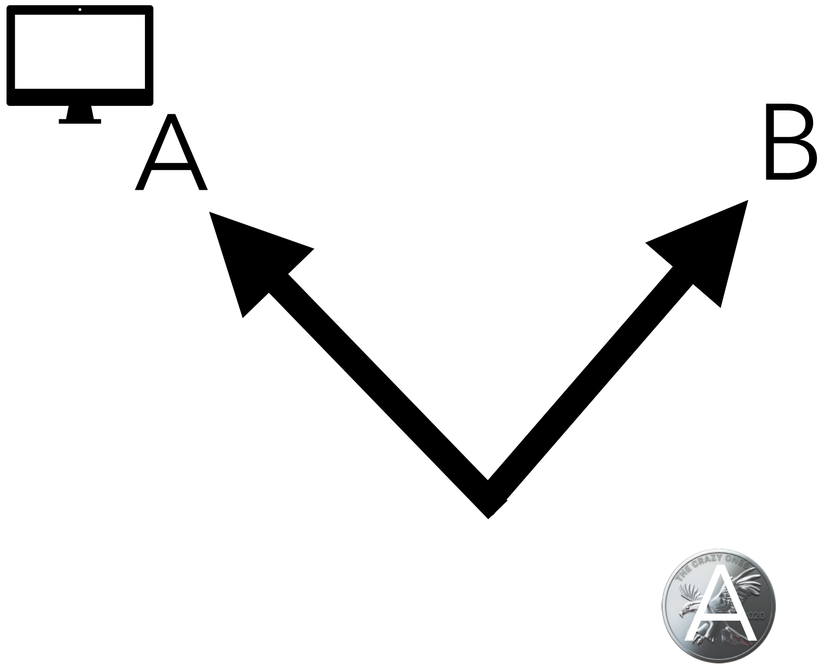
Topics Overview

Randomized Algorithm



Topics Overview

Randomized Algorithm



Tools for Understanding Why This Works

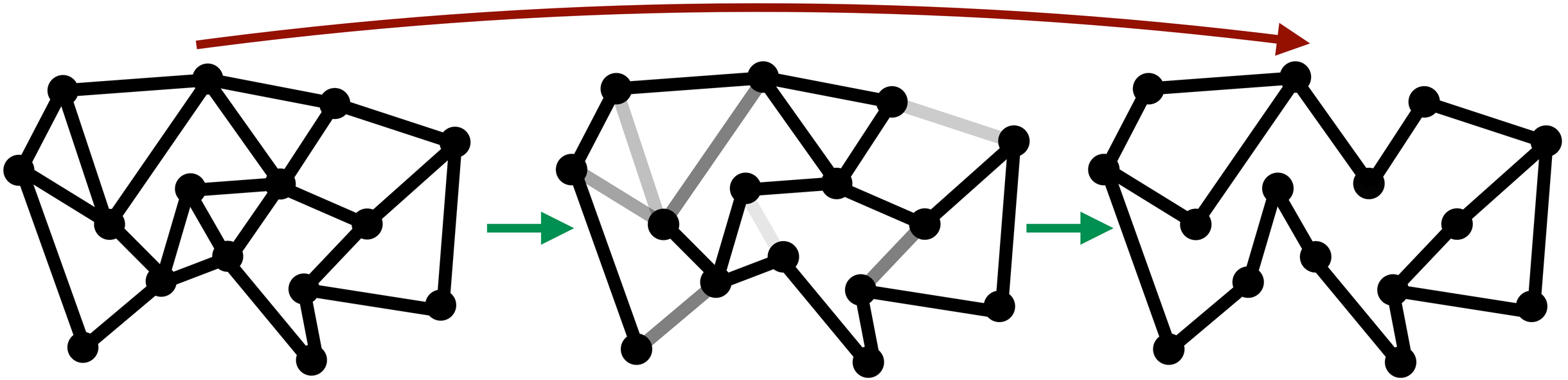
Topics Overview

Polyhedral Methods

Making a problem **continuous** makes it easier

Topics Overview

Polyhedral Methods



Input Problem *Fractional Solution* *Solution*

How to Solve Problems by First Fractionally Solving Them

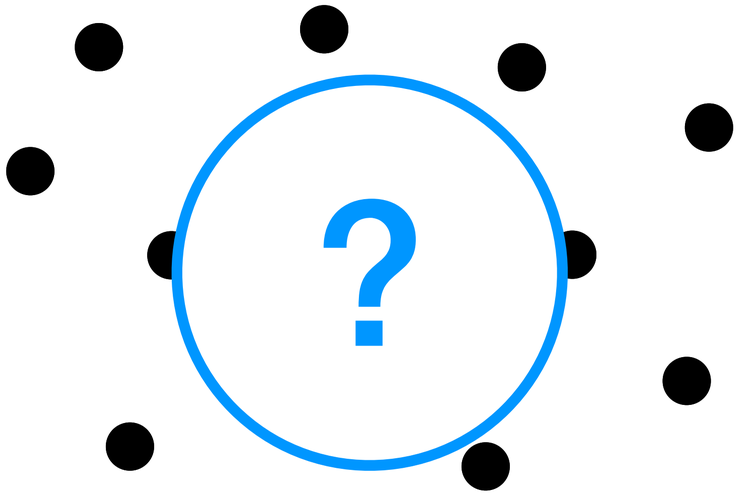
Topics Overview

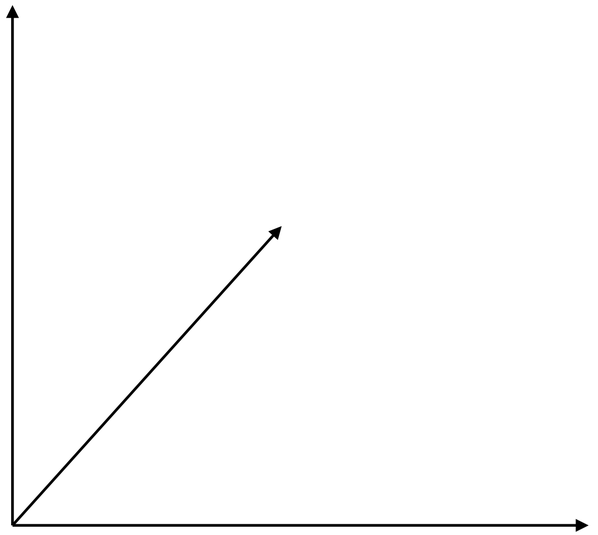
Geometry and Metric Embeddings

Simplifying the **geometry** of a problem makes it easier

Topics Overview

Geometry and Metric Embeddings



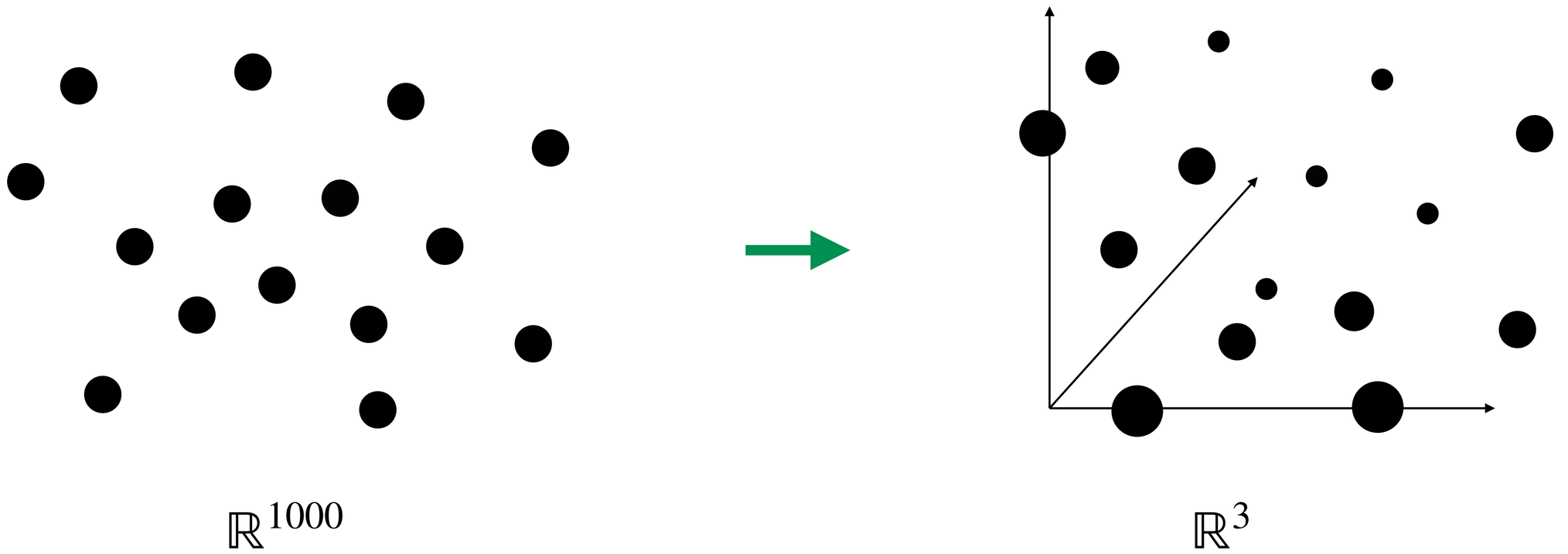


$\mathbb{R}^{1000}$

$\mathbb{R}^{3}$

Topics Overview

Geometry and Metric Embeddings



Tools for Reasoning About and Simplifying Metrics

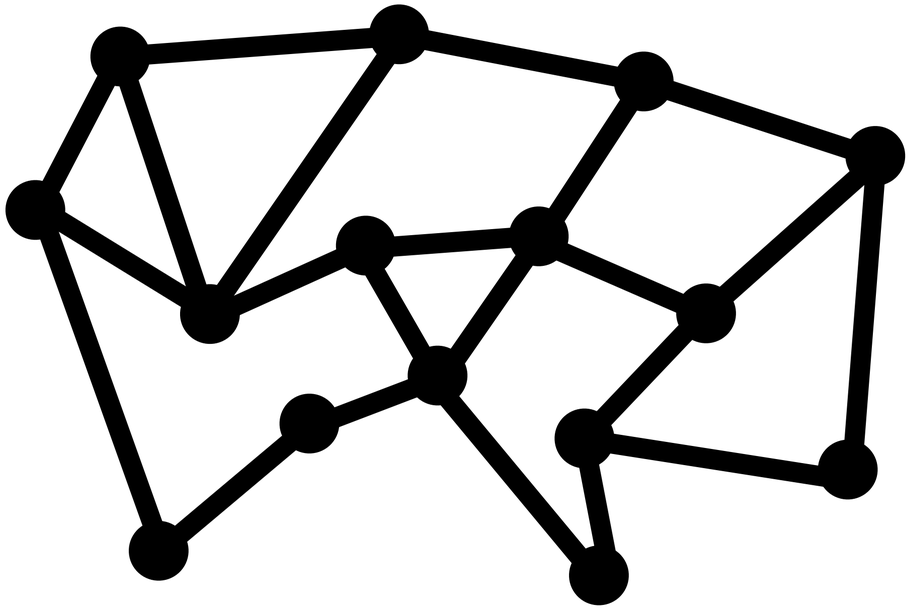
Topics Overview

Cuts and Flows

Max **flow** = min **cut** makes a lot of problems easier

Topics Overview

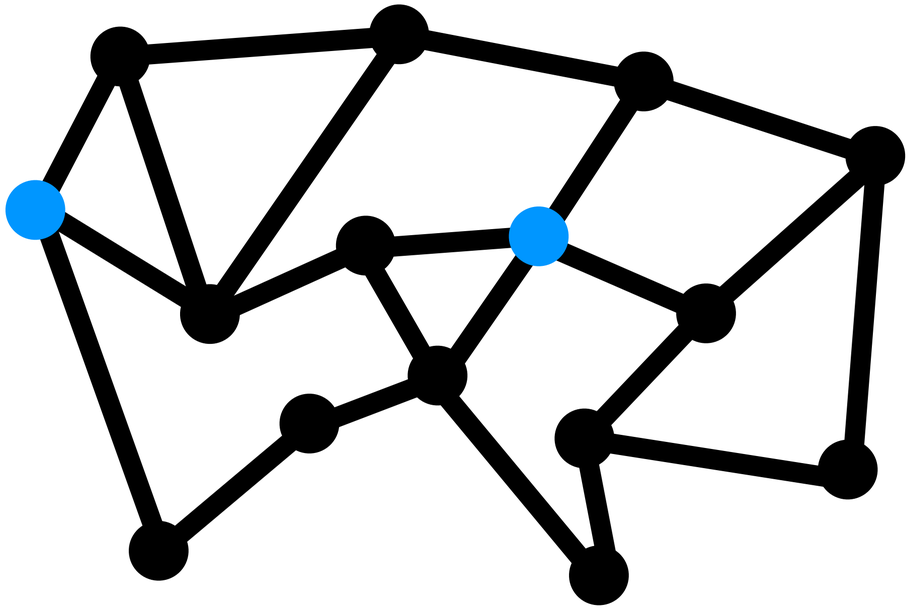
Cuts and Flows



Max Flow

Topics Overview

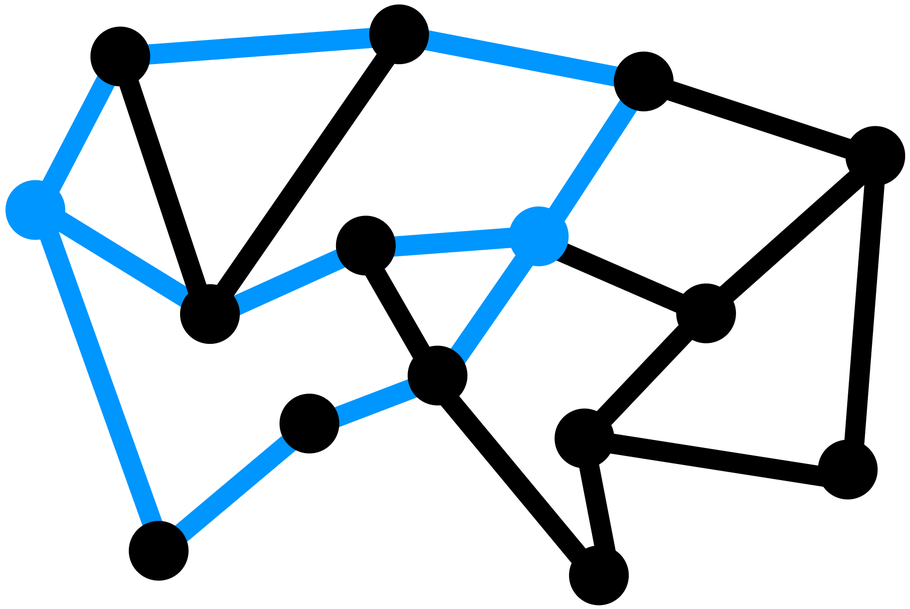
Cuts and Flows

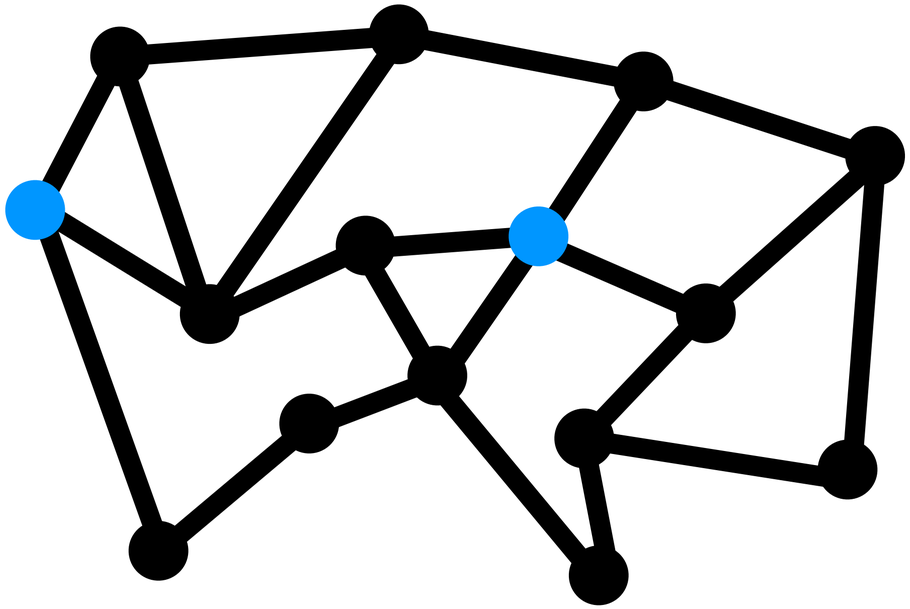


Max Flow

Topics Overview

Cuts and Flows

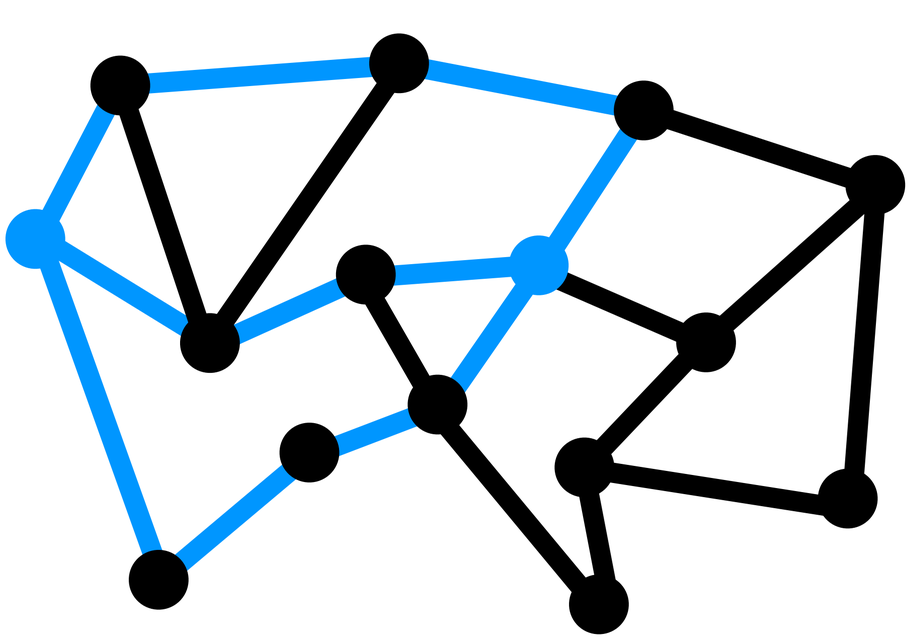


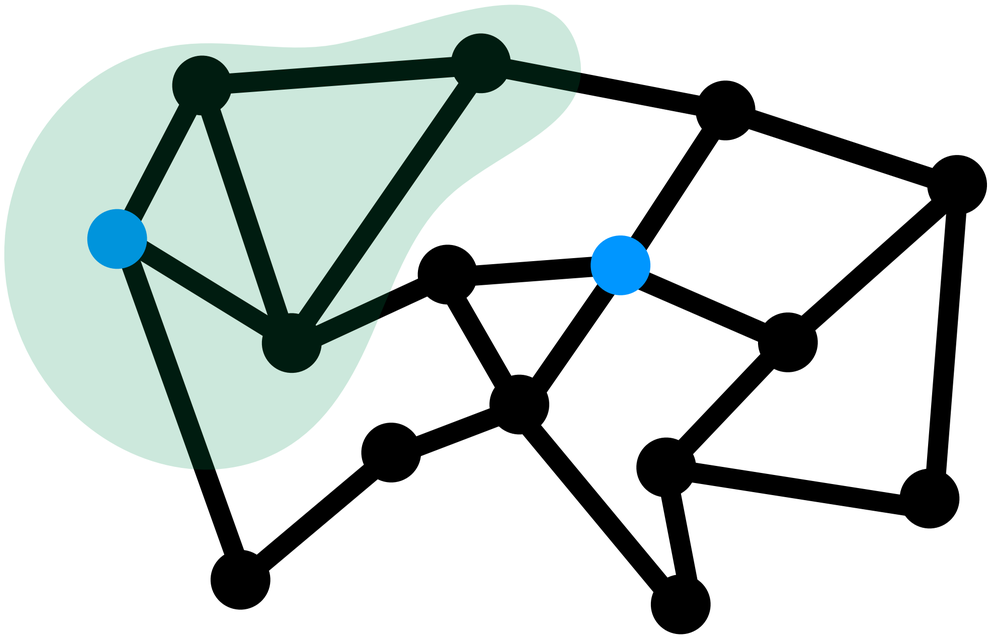


Max Flow = *Min Cut*

Topics Overview

Cuts and Flows

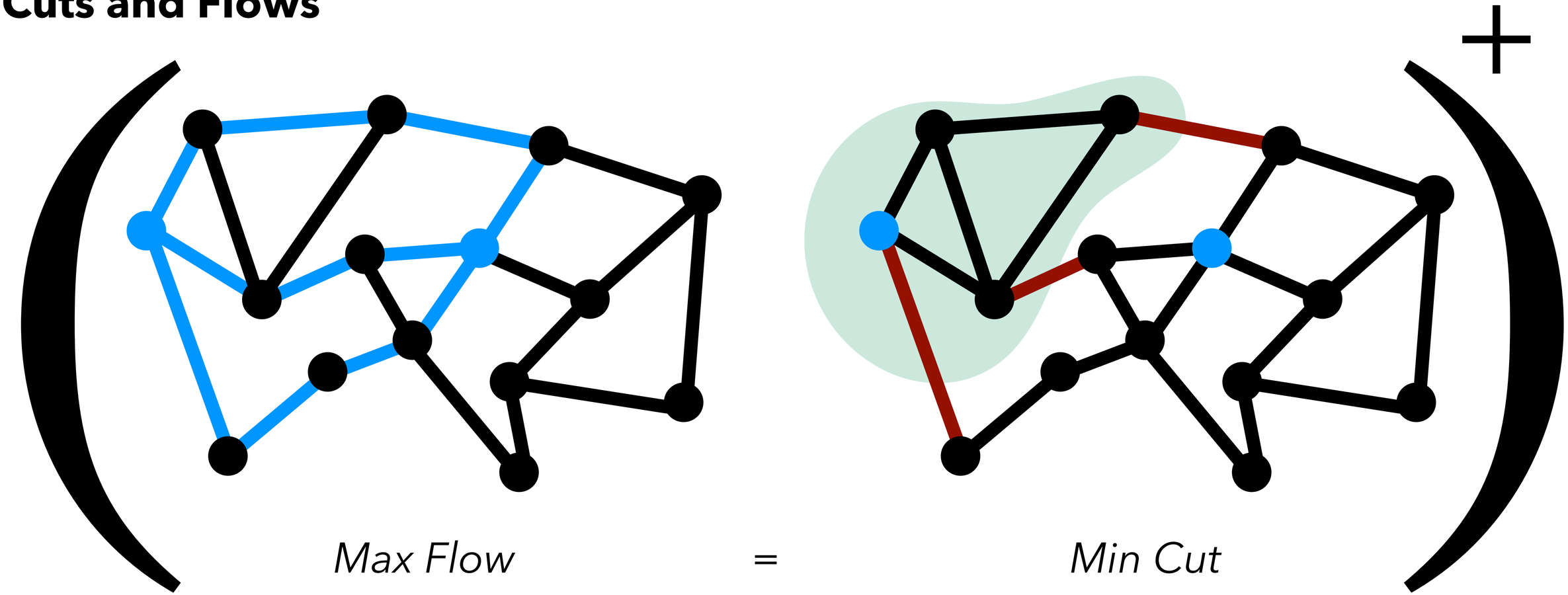




Max Flow = *Min Cut*

Topics Overview

Cuts and Flows



Max Flow = *Min Cut*

Strengthening and Generalizing Flow/Cut Machinery

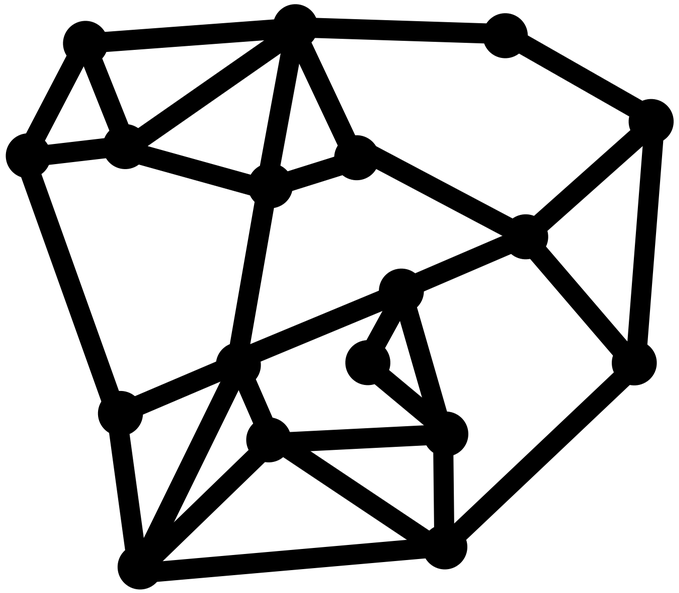
Topics Overview

Graph Sparsification

Simplifying **graph** structure makes graph problems easier

Topics Overview

Graph Sparsification

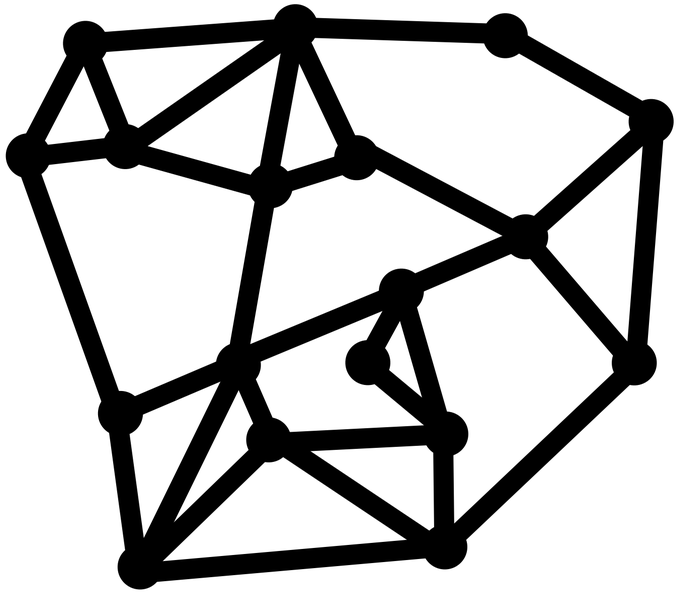


graph $G = (V, E)$

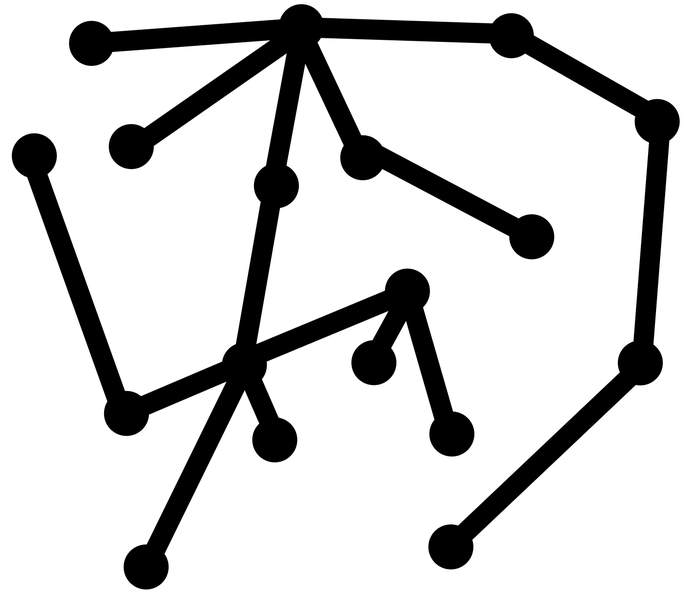
simple representation H
of some property of G

Topics Overview

Graph Sparsification



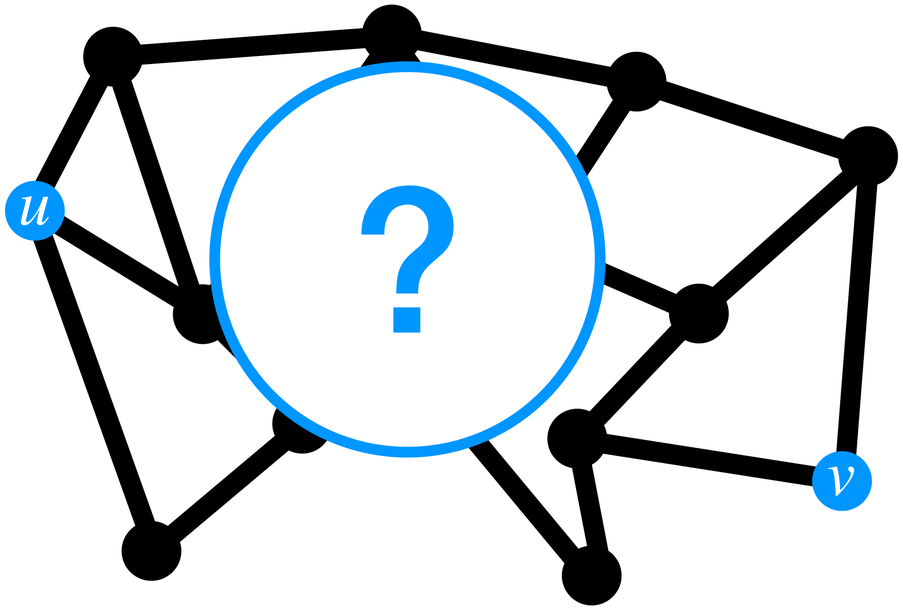
graph $G = (V, E)$

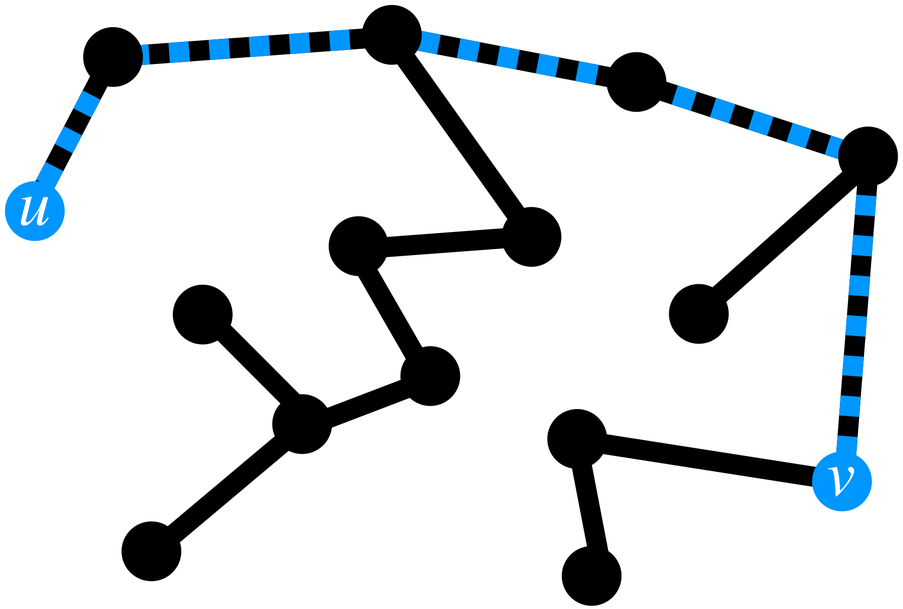


spanning *tree* H
s.t. $d_G = d_H$

Topics Overview

Graph Sparsification





What's the $u \to v$ shortest path?

What's the $u \to v$ shortest path?

Methods for Sparsifying Graphs

Topics Overview

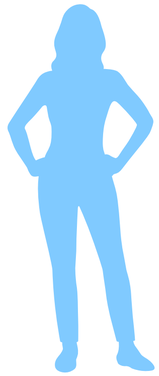
Multiplicative Weights

Deferring to **experts** makes decisions easier

Topics Overview

Multiplicative Weights

Buy or Sell?



You

Should buy

Topics Overview

Multiplicative Weights

Buy or Sell?

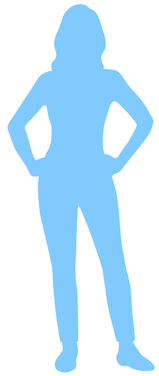
You

Should sell

Topics Overview

Multiplicative Weights

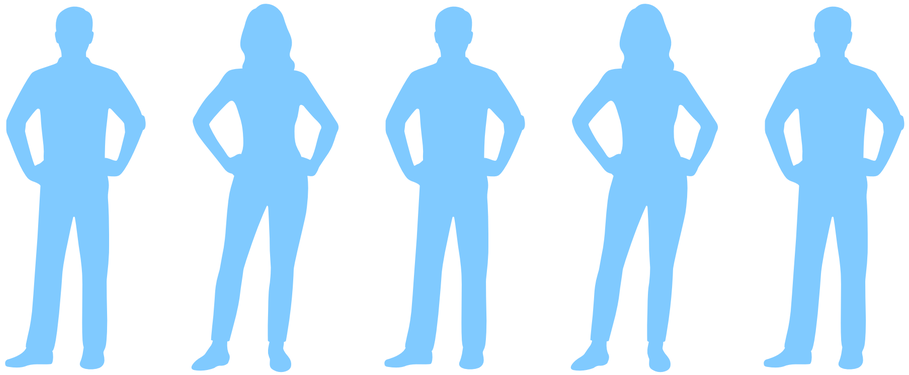
Buy or Sell?



You

Buy =

Buy *Sell* *Buy* *Buy* *Sell*

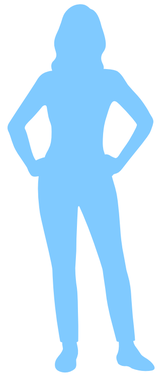


Experts

Topics Overview

Multiplicative Weights

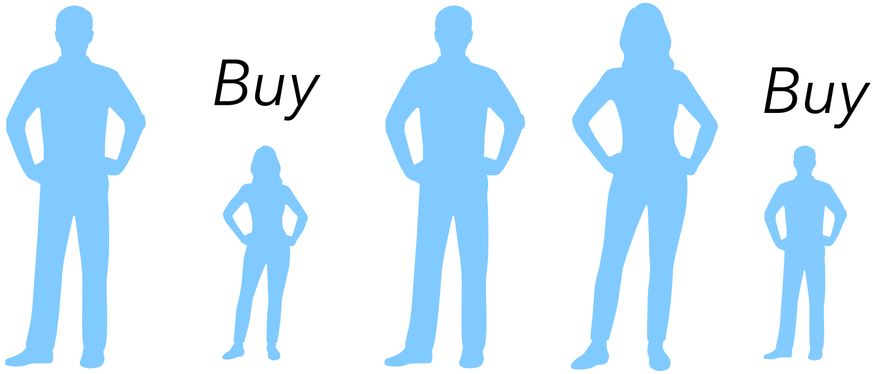
Buy or Sell?



You

Sell =

Buy *Buy* *Sell* *Sell* *Buy*

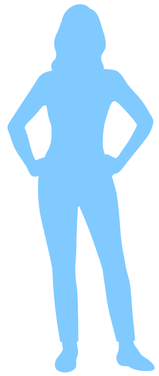


Experts

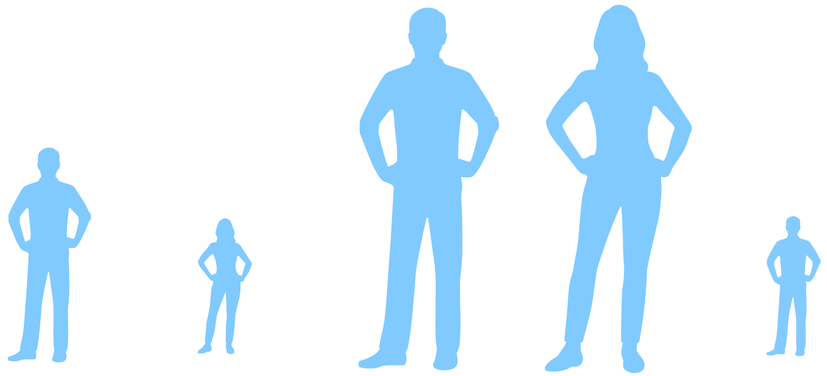
Topics Overview

Multiplicative Weights

Buy or Sell?



You

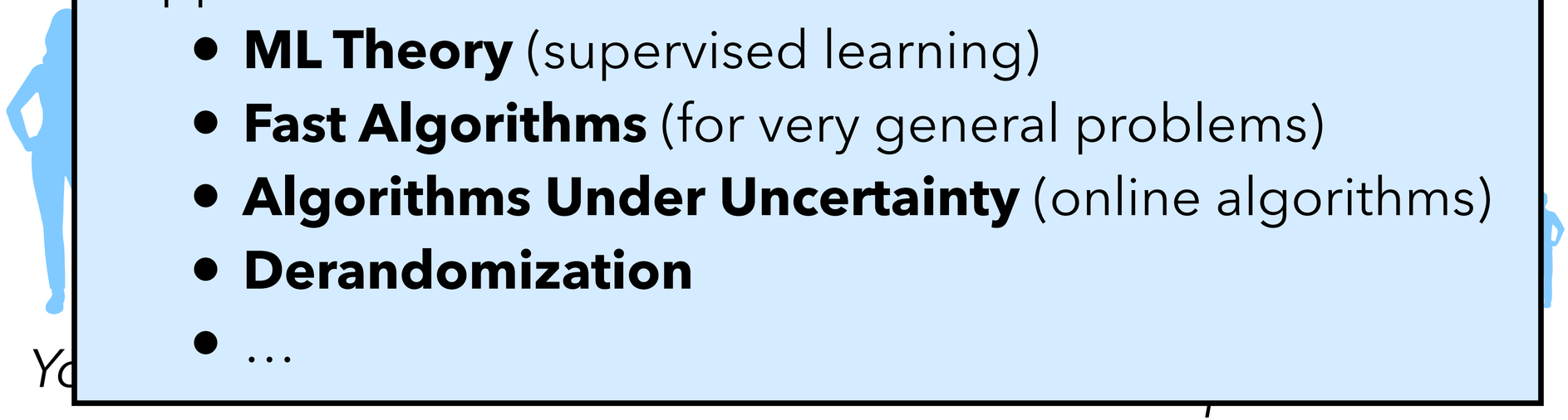


Experts

Over time perform ≈ as well as best expert

Topics Overview

Multiplicative Weights



Applications in:

- **ML Theory** (supervised learning)
- **Fast Algorithms** (for very general problems)
- **Algorithms Under Uncertainty** (online algorithms)
- **Derandomization**
- …

Over time perform ≈ as well as best expert

Class Logistics

Class Logistics

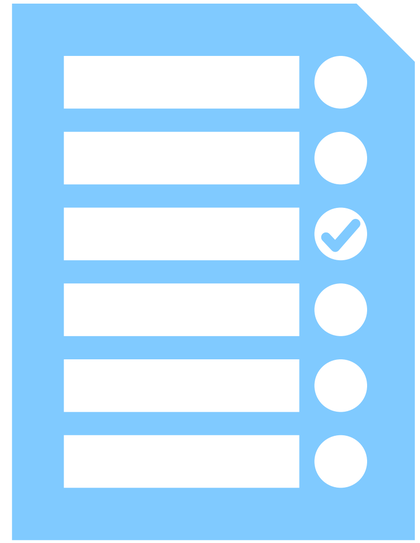
Your Responsibilities and Grade Breakdown

- 6 homeworks (60% total, 10% each, one topic each)
- 1 final (25% total)
- Theory Seminar participation twice (10%, 5% per seminar)
- Class participation (10%)

Class Logistics

Grade Distribution

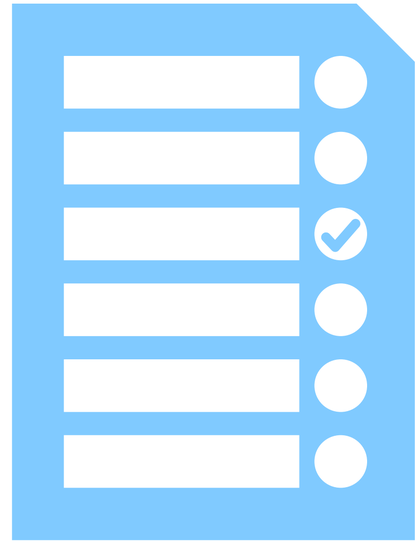
- Grades:
  - A (≥90%)
  - B (≥80%)
  - C (≥70%)
  - NC (<70%)
- May curve upward



Class Logistics

Misc. Logistics (1/3)

- All other classes are board talks; 1 or 2 may be subbed
- Homeworks can be collaborative (up to 4)
- First homework out this week
- Final is take-home, not collaborative
- All assignments must be written in LaTex
- Submissions via Gradescope

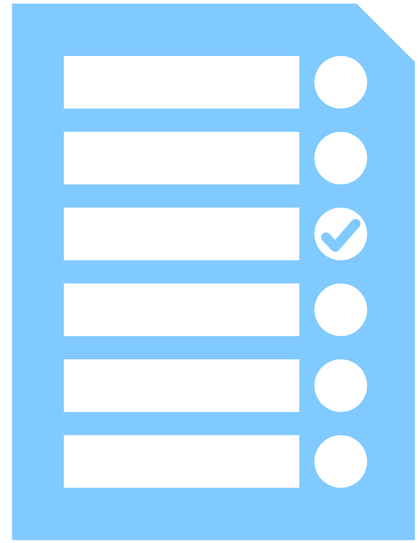


Class Logistics

Misc. Logistics (2/3)

- For each Theory Seminar: submit a short summary (~150 words) of what you learned / how it relates to anything in which you're interested
- Goal is for class to be maximally interactive so please, please, (please):
  - Ask questions if you're confused
  - Do your best to answer questions, even if unsure
- Course website:

  https://dhershko.github.io/teaching/fall24Toolkit.html



Class Logistics

Misc. Logistics (3/3)

- TAs:
  - Jay Sarva (UTA)
  - Richard Huang (grad TA)
- Office hours:
  - Jay: 4PM-6PM Tuesdays in CIT 227
  - Ellis: 1:30PM-2:30PM Wednesdays in CIT 507
  - Richard: 3PM-4PM Fridays in CIT 361